2. PROBLEM DESCRIPTION

$\dot{x} = Ax + B_1 u_1 + B_2 u_2$

$y = Cx$

$x \in \mathbb{R}^n$

$u_1 \in \mathbb{R}^{m_1}$

$u_2 \in \mathbb{R}^{m_2}$

$y \in \mathbb{R}^p$

$$J(x(0), u_1, u_2) = \int_0^\infty \left(y^T Q_y y + u_1^T u_1 - \gamma^2 u_2^T u_2\right) d\tau$$

$Q_y \geq 0$

$\gamma > 0$

$$\int_0^\infty \left(y^T Q_y y + u_1^T u_1\right) d\tau \leq \gamma^2 \int_0^\infty u_2^T u_2 \, d\tau$$

$u_2 \in l_2[0, \infty)$

(u_1^*, u_2^*)

$J(x(0), u_1^*, u_2) \leq J(x(0), u_1^*, u_2^*) \leq J(x(0), u_1, u_2^*)$

$u_1(x), u_2(x)$

$u_1^* = k^* x$

$u_2^* = L^* x$

$\sqrt{Q_x}^T \sqrt{Q_x} = Q_x \, , Q_x = C^T Q_y C$

$k^* = -B_1^T P^*$

$L^* = \gamma^{-2} B_2^T P^*$

P^*

$$A^T \mathrm{P} + \mathrm{PA} + \mathrm{Q} - \mathrm{P}(B_1 B_1^T - \gamma^{-2} B_2 B_2^T)P = 0$$

3. OUTPUT FEEDBACK DESIGN FOR LINEAR DIFFRENTIAL ZERO-SUM GAMES.

THEORIM 1.

$$\bar{x} = M_1\omega_1 + M_2\omega_2 + M_3\omega_3$$

$$M_j^i = \begin{bmatrix} a_{j0}^{i1} & a_{j1}^{i1} & \cdots & a_{j(n-1)}^{i1} \\ a_{j0}^{i2} & a_{j1}^{i2} & \cdots & a_{j(n-1)}^{i2} \\ \vdots & \vdots & \vdots & \vdots \\ a_{j0}^{in} & a_{j1}^{in} & \cdots & a_{j(n-1)}^{in} \end{bmatrix}$$

$$i \in \{1,2,\dots,m_1\},\{1,2,\dots,m_2\},\{1,2,\dots,p\}, for\ j = 1,2,3$$

$$\dot{\omega}_1^i = A\omega_1^i + bu_1^i, \qquad \omega_1^i(0) = 0, i = 1,2,\dots,m_1$$

$$\dot{\omega}_2^i = A\omega_2^i + bu_2^i, \qquad \omega_2^i(0) = 0, i = 1,2,\dots,m_2$$

$$\dot{\omega}_3^i = A\omega_3^i + bu_3^i, \qquad \omega_2^i(0) = 0, i = 1,2,\dots,p$$

$$A = \begin{bmatrix} 0 & 1 & 0 & & \dots & 0 \\ 0 & 0 & 1 & & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & & \vdots \\ 0 & 0 & 0 & \dots & & 1 \\ -\alpha 0 & -\alpha 1 & \dots & \dots & & -\alpha_{n-1} \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

PROOF.

$\hat{x}$

$$\dot{\hat{x}} = A\hat{x} + B_1u_1 + B_2u_2 + F(y - C\hat{x}) = (A - FC)\hat{x} + B_1u_1 + B_2u_2 + F_y$$

$$\hat{x} = (sI - A + FC)^{-1}B_1[u_1] + (sI - A + FC)^{-1}B_2[u_2] + (sI - A + FC)^{-1}F[y] + e^{(A-FC)t}\hat{x}(0)$$

$$\hat{x} = \sum_{i=1}^{m_1} \frac{U_1^i(s)}{\Lambda(s)}[u_1^i] + \sum_{i=1}^{m_2} \frac{U_2^i(s)}{\Lambda(s)}[u_2^i] + \sum_{i=1}^{p} \frac{Y^i(s)}{\Lambda(s)}[y^i] + e^{(A-FC)t}\hat{x}(0)$$

u_1^i

u_2^i

$U_1^i(s)$

$U_2^i(s)$

$Y^i(s)$

$s[x] = \frac{d}{dt} x$

$\Lambda(s)$

$\Lambda(s) = det(sI - A + FC) = s^n + \alpha_{n-1} s^{n-1} + \alpha_{n-2} s^{n-2} + \cdots + \alpha_1 s + \alpha_0$

$\frac{U_1^i(s)}{\Lambda(s)} [u^i]$

$\frac{U_1^i(s)}{\Lambda(s)}$

$$\frac{U_1^i(s)}{\Lambda(s)} [u_1^i] = \begin{bmatrix} \frac{a_{1(n-1)}^{i1} s^{n-1} + a_{1(n-2)}^{i1} s^{1(n-2)} + \cdots + a_{10}^{i1}}{s^n + \alpha_{n-1} s^{n-1} + \alpha_{n-2} s^{n-2} + \cdots + \alpha_0} \\ \frac{a_{1(n-1}^{i2} s^{n-1} + a_{1(n-2)}^{i2} s^{n-2} + \cdots + a_{10}^{i2}}{s^n + \alpha_{n-1} s^{n-1} + \alpha_{n-2} s^{n-2} + \cdots + \alpha_0} \\ \vdots \\ \frac{a_{1(n-1)}^{in} s^{n-1} + a_{1(n-2)}^{in} s^{n-2} + \cdots + a_{10}^{in}}{s^n + \alpha_{n-1} s^{n-1} + \alpha_{n-2} s^{n-2} + \cdots + \alpha_0} \end{bmatrix} [u_1^i]$$

$$= \begin{bmatrix} a_{10}^{i1} & a_{11}^{i1} & \cdots & a_{1(n-1)}^{i1} \\ a_{10}^{i2} & a_{11}^{i2} & \cdots & a_{1(n-1)}^{i2} \\ \vdots & \vdots & \vdots & \vdots \\ a_{10}^{in} & a_{11}^{in} & \cdots & a_{1(n-1)}^{in} \end{bmatrix} \triangleq M_1^i \omega_1^i$$

$M_1^i \in \mathbb{R}^{n \times n}$

$\omega_1^i \in \mathbb{R}^n$

$\dot{\omega}_1^i = A \omega_1^i + b u_1^i, \;\; \omega_1^i(0) = 0$

$I = 0,1, \dots, m_1$

$\omega_1 = \left[(\omega_1^1)^T (\omega_1^2)^T \dots \left(\omega_1^{m_1}\right)^T\right]^T \in \mathbb{R}^{m_1 n}$

$M_1 = \left[M_1^1 M_1^2 \dots M_1^{m_1}\right] \in \mathbb{R}^{n \times m_1 n}$

$\dot{\omega}_2^i = A\omega_2^i + bu_2^i, \qquad \omega_2^i(0) = 0$

$\dot{\omega}_3^i = A\omega_3^i + bu_3^i, \qquad \omega_2^i(0) = 0$

$u_2^i, i = 0,1, \dots, m_2$

$y^i, i = 0,1, \dots, p$

$\hat{x} = M_1\omega_1 + M_2\omega_2 + M_3\omega_3 + e^{(A-FC)t}\hat{x}(0)$

$e = x - \hat{x} = e^{(A-FC)t}e(0)$

$x = M_1\omega_1 + M_2\omega_2 + M_3\omega_3 + e^{(A-FC)t}x(0)$

Remark 1.

$\acute{\omega}_J s$

$\acute{M}_J s$

a_{jl}^{ik}

$\frac{U_1}{\Lambda(s)}, \frac{U_2}{\Lambda(s)}, \frac{Y}{\Lambda(s)}$

(A, B_1, B_2, C, F)

Proof.

$$(sI-(A-FC))^{-1}f[y]=\frac{D_{n-1}s^{n-1}+D_{n-2}s^{n-2}+\cdots+D_0}{\alpha_{n-1}s^{n-1}+\alpha_{n-2}s^{n-2}+\cdots+\alpha_0}F[y]$$

$$=[D_0F\quad D_1F\quad \dots\quad D_{n-1}F]\begin{bmatrix}\frac{1}{\Lambda(s)}[y]\\ \frac{s}{\Lambda(s)}[y]\\ \vdots\\ \frac{s^{n-1}}{\Lambda(s)}[y]\end{bmatrix}\triangleq M_3\omega_3$$

$$D_{n-1}=I$$

$$D_{n-2}=(A-FC)+\alpha_{n-1}I$$

$$D_{n-3}=(A-FC)^2+\alpha_{n-1}(A-FC)+\alpha_{n-2}I$$

$$\vdots$$

$$D_0=(A-FC)^{n-1}+\alpha_{n-1}(A-FC)^{n-2}+\cdots+\alpha_2(A-FC)+\alpha_1I$$

D_i, M_3

$$\rho(M_3)=\rho([(A-FC)^{n-1}F+\alpha_{n-1}(A-FC)^{n-2}F\dots+\alpha_2(A-FC)F+\alpha_1F,\dots,(A-FC)F+\alpha_{n-1}F,F])$$

$$\rho(M_3)=\rho([(A-FC)^{n-1}F,\dots,(A-FC)F,F])$$

$(\mathrm{A}-\mathrm{FC},\mathrm{F})$

$q^T[A - FC - \lambda I, \quad F] = 0$

$q^T(A - FC) = \lambda q^T, q^T F = 0, \;\; q^T \mathrm{A} = \lambda q^T$

$let z = [\omega_1^T \omega_2^T \omega_3^T]^T \in \mathbb{R}^N$

$M = [M_1 M_2 M_2] \in \mathbb{R}^{n\times N}$

$N = m_1 n + m_2 n + pn$

V^*

$x^T p^* x$

$\bar{p}^* = M^T p^* M$

$\bar{V}^* = z^T \bar{p}^* z$

$u_1^* = k^* x = K^* Mz + k^* e^{(A-FC)t} x(0)$

$u_2^* = L^* x = L^* Mz + L^* e^{(A-FC)t} x(0)$

$\bar{u}_1^* = \bar{k}^* z$

$\bar{u}_2^* = \bar{L}^* z$

$\bar{K}^* = K^* M \in \mathbb{R}^{m_1 \times N}$

$\bar{L}^* = L^* M \in \mathbb{R}^{m_2 \times N}$

$$x^T(t) P^i x(t) - x^T(t-T) P^i x(t-T) = -\int_{t-T}^{t} x^T(\tau)\left(Q_x + \left(K^i\right)^T\left(K^i\right) - \gamma^2\left(L^i\right)^T\left(L^i\right)\right) x(\tau) d\tau - 2\int_{t-T}^{t}\left(u_1(\tau) - K^i x(\tau)\right)^T K^{i+1} x(\tau) d\tau + 2\gamma^2 \int_{t-T}^{t}\left(u_2(\tau) - L^i x(\tau)\right)^T L^{i+1} x(\tau) d\tau$$

$K^{i+1} = -B_1^T P^i \,, \;\; L^{i+1} = \gamma^{-2} B_2^T P^i$

$$x = Mz + e^{(A-FC)t}x(0), \quad x^T Q_x x = y^T Q_y y$$

$$\begin{aligned}
&\left(Mz(t) + e^{(A-FC)t}x(0)\right)^T P^i \left(Mz(t) + e^{(A-FC)t}x(0)\right) \\
&\qquad - \left(Mz(t-T) + e^{(A-FC)(t-T)}x(0)\right)^T P^i \\
&\qquad \times \left(Mz(t-T) + e^{(A-FC)(t-T)}x(0)\right) \\
&\qquad = -\int_{t-T}^{t} y^T(\tau) Q_y(\tau) d\tau - \int_{t-T}^{t} \left(Mz(\tau) + e^{(A-FC)(\tau)}x(0)\right)^T \\
&\qquad \times \left(K^i\right)^T \left(K^i\right) \left(Mz(\tau) + e^{(A-FC)(\tau)}x(0)\right) d\tau \\
&\qquad + \gamma^2 \int_{t-T}^{t} \left(Mz(\tau) + e^{(A-FC)(\tau)}x(0)\right)^T \\
&\qquad \times \left(L^i\right)^T \left(L^i\right) \left(Mz(\tau) + e^{(A-FC)(\tau)}x(0)\right) d\tau \\
&\qquad - 2\int_{t-T}^{t} \left(u_1(\tau) - K^i \left(Mz(\tau) + e^{(A-FC)(\tau)}x(0)\right)\right)^T \\
&\qquad \times K^{i+1} \left(Mz(\tau) + e^{(A-FC)(\tau)}x(0)\right) d\tau \\
&\qquad + 2\gamma^2 \int_{t-T}^{t} \left(u_2(\tau) - L^i \left(Mz(\tau) + e^{(A-FC)(\tau)}x(0)\right)\right)^T \\
&\qquad \times L^{i+1} \left(Mz(\tau) + e^{(A-FC)(\tau)}x(0)\right) d\tau.
\end{aligned}$$

تا قسمت اول صفحه ی ۴ نوشته شد.